Technical Notes

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Reversed Flow Above a Plate with Suction

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Introduction

MOST of the known exact analytical solutions of the Navier-Stokes equations have been obtained for parallel laminar flows for which the Navier-Stokes equations can be linearized. Taylor[1] observed that the nonlinear convective terms in the two-dimensional Navier-Stokes equations vanish when the vorticity is a function of the Stokes stream function alone. For the special case of vorticity proportional to the stream function, he obtained an exact solution that represented a double infinite array of vortices decaying exponentially with time. Kovasznay[2] extended Taylor's method, and considered the flow in which the local vorticity is proportional to the stream function perturbed by a uniform stream. He was able to linearize the Navier-Stokes equation and obtained an exact solution which may be used to represent the flow downstream of a two-dimensional grid.

In this Note we make a minor extension of Kovasznay's solution, and present an exact solution of the Navier-Stokes equations that may represent the reversed flow about a flat plate with suction. The present exact solution grew out of our search for a simple exact solution that may serve as an exact basic flow for a model stability analysis of nonparallel flows involving a flow reversal. Unlike the stability analysis of parallel flows, the stability analysis of nonparallel flows is not yet firmly established. Among other difficulties, the stability analysis of nonparallel flows suffers from a serious uncertainty associated with the approximate nature of nonparallel basic flows.

Exact Solution

Consider two-dimensional flows of an incompressible Newtonian fluid. The X and Z Cartesian components of the velocity field are related respectively to the Stokes stream function Ψ by

$$U=\Psi_Z, \quad V=-\Psi_X$$

where subscripts X and Z denote partial differentiations. In terms of Ψ, the governing equation is

$$(\partial_t - \nu\underline{\nabla}^2)\underline{\nabla}^2\Psi = \Psi_X\underline{\nabla}^2\Psi_Z - \Psi_Z\underline{\nabla}^2\Psi_X \tag{1}$$

where t is time, ν the kinematic viscosity, $\underline{\nabla}^2$ the Laplacian operator, and X and Z denote partial differentiations with respect to the space variables. By using the following dimensionless variables

$$\psi = \frac{\Psi}{\nu}, \quad (x,z) = \frac{(X,Z)}{(\nu/U)}, \quad \nabla = \underline{\nabla}\left(\frac{\nu}{U}\right), \quad \tau \frac{t}{(\nu/U^2)}$$

where U is the characteristic velocity, Eq. (1) can be rewritten as

$$(\partial_\tau - \nabla^2)\nabla^2\psi = \psi_x\nabla^2\psi_z - \psi_z\nabla^2\psi_x \tag{2}$$

This nonlinear equation can be linearized for flows with the following particular vorticity distribution

$$\nabla^2\psi = k(\psi - Rz) \tag{3}$$

where k is a constant to be determined and R a flow parameter to be explained. Substitution of Eq. (3) into Eq. (2) gives the following linear equation

$$(\partial_\tau - \nabla^2)\psi = -R\psi_x \tag{4}$$

A special steady solution, ψ, of Eq. (4) is given by

$$\psi = Rz + B\exp(mx - nz) \tag{5}$$

where B and n are constants, and

$$m = \tfrac{1}{2}(R\sqrt{R^2 - 4n^2}) \tag{6}$$

The coefficient of vorticity distribution k appearing in Eq. (3) can now be determined by substituting Eqs. (5) and (6) into Eq. (3). It is found that $k = mR$.

When $n = 2\pi i$ and R is identified as the Reynolds number, the solution given by Eqs. (5) and (6) reduces to the Kovasznay flow. Kovasznay suggested that the solution corresponding to the negative root of m in Eq. (6), i.e., $m = 0.5 \times (R - \sqrt{R^2 + 16\pi^2})$, can be used to describe the wake flow in the region $x > 0$ behind a grid at $x = 0$. The solution corresponding to the positive root of m, i.e., $m = 0.5 \times (R + \sqrt{R^2 + 16\pi^2})$, can be used to describe the two-dimensional uniform flow, in $x < 0$, perturbed by a grid of blunt bodies with suction or blowing whose magnitude is proportional to B near the front stagnation points.

When n is real, the new exact solution given by Eqs. (5) and (6) represent flows over a plate with suction or blowing of fluid at the plate. For example, when $n > 0$ and $R^2 - 4n^2 > 0$ the exact solution describes the flow over a porous plate $z = 0$, $x < 0$ with velocity components given by

$$u = \psi_z = R - nB\ \exp\ (mx - nz) \tag{7}$$

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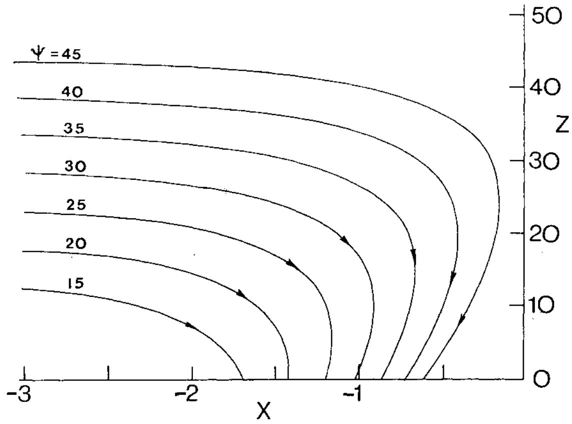

Fig. 1 Streamline pattern for $R=1$, $B=80$, $C=0.05$.

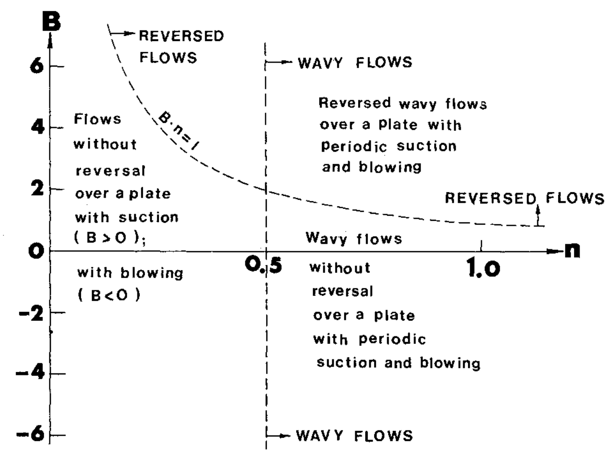

Fig. 2 Exact solutions of the Navier-Stokes equations for flows over a plate, $z=0$, $x<0$.

$$v = -\psi_x = -mB \exp(mx - nz) \tag{8}$$

It is seen from Eqs. (7) and (8) that R can be set equal to 1 without loss of generality, since it amounts to rescaling the velocity by a factor of R. It is seen from Eq. (8) that $B>0$ corresponds to suction and $B<0$ corresponds to blowing at the bottom plate, $z=0$. Figure 1 gives the streamlines of the flow for $R=1$, $n=0.05$, and $B=80$. Note the flow reversal due to suction near the trailing edge. It should be pointed out that the flow reversal may not take place when B is sufficiently small.

When $n>0$ but $R^2-4n^2=-q^2<0$, the real part of Eq. (5) yields

$$\psi = Rz + B\exp(Rx/2 - nz)\cos qx$$

Hence

$$u = R - nB\exp(Rx/2 - nz)\cos qx$$

$$v = -B[(R/2)\cos qx - q\sin qx]\exp(Rx/2 - nz)$$

This solution represents the wavy flow over a plate $z=0$, $x<0$ with a periodic variation of suction and blowing along the plate. The possible flows represented by Eqs. (5) and (6), with n real, are delineated in Fig. 2. Note that $R=1$ without loss of generality.

Discussion

A new exact solution of the Navier-Stokes equations is given. The solution can be used to describe the flow with or without flow reversal above a plate with suction and/or blowing. Aside from its own intrinsic interest, it provides us with a simple but exact nonparallel basic flow which could be used for a model analysis of stability of nonparallel flow. The stability analysis of nonparallel flows usually suffers from the uncertainty associated with modeling the nonparallel flows with approximate solutions.

Acknowledgments

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References

[1]Taylor, G. I., "On the Decay of Vorticies in a Viscous Fluid," *Philosophical Magazine*, Vol. 46, 1923, pp. 671-4.

[2]Kovasznay, L.I.G., "Laminar Flow Behind a Two-Dimensional Grid," *Proceedings of the Cambridge Philosophical Society*, Vol. 44, 1948, pp. 58-62.

Effect of a Nearby Solid Surface on a Five-Hole Pressure Probe

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Introduction

HIGH sensitivity and resolution levels have made multiple-hole probes reliable instruments to use in determining rather complex flow situations. In particular, five-hole probes have been used to analyze flow properties of high flow angle regions generated by lift enhancement devices, such as canards, of three-dimensional flows inside compressors and turbines, and of concentric swirling flows inside mixers.

Such probes must be carefully calibrated, however, and calibration usually is carried out in a well-understood uniform stream. When solid boundaries are present, substantial flow realignment can occur with resultant change in measured pressures, so that multiple-hole probes have to be restricted in their use to flows far from containing walls.

In the present investigation, two geometrically similar probes of substantially different scales were first calibrated and then tested in the vicinity of a solid surface. The results provided data on the magnitude of the wall influence and, in addition, indicate, that by including the effect of surface interaction in the calibrating procedure, the range of validity of the results can be slightly extended.

Experimental Procedure

Two probes were used throughout this study. The first probe had an external diameter of 0.158 in., the second had a diameter of 0.6 in. Their geometries were similar in every point (i.e., 45-deg cone angle at the tip and pressure ports located at midspan of the conical surface), except that the pressure ports were not scaled.

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